Appendix 2

Significant Figures

2.4 Objective

In this module, methods for reporting laboratory data having limitations to their accuracy are introduced.

2.5 Introduction

In science there are two methods of acquiring numbers. One method is by counting; the other is by measurement. Counting is by its very nature exact. Measurement on the other hand is done by comparison with a calibrated instrument. Since there are limitations on the scales of all calibrated instruments, there is a limit to the accuracy to which any measurement may be made. The use of significant figures expresses the extent of this accuracy. The use of significant figures is really only an approximate method for handling uncertainty in measurement and its propagation.

Generally the number of significant figures to which a measurement should be reported includes <u>all of those digits which are certain and only one digit, the last, which is uncertain</u>. Thus, for example, if one were to measure the width of a page of paper using a 30 cm ruler graduated to the nearest $0.1\ cm$, one might find that it measures somewhere between $21.6\ cm$ and $21.7\ cm$. Furthermore, one may estimate that this width lies approximately halfway between the two and therefore record the width as $21.65\ cm$. In this instance, the last $0.05\ cm$ is uncertain.

The following set of rules is helpful in determining which of the digits in a number are significant.

2.6 Determining the Number of Significant Figures

Non-zero Digits
All non-zero digits in a number are significant.

Zeroes
There are three classes of zeroes to be considered

a) Leading Zeroes – never significant
   e.g. 0001234 – 4 significant figures, one for each non-zero digit.
   0.001234 – 4 significant figures, one for each non-zero digit.

b) Trapped Zeroes – always significant
   e.g. 1001 – 4 significant figures; the zeroes are trapped and are therefore significant

c) Trailing Zeroes – may or may not be significant
   If a decimal point exists somewhere in the number, the trailing zeroes are significant.
   e.g. 10.0 – 3 significant figures; a decimal points exists.
   0.0102000 – 6 significant figures; the leading zeroes are not significant, the trapped zero is and since there is a decimal point in the number, the trailing zeroes are significant.

   If there is no decimal point anywhere in the number, the situation is ambiguous.

e.g.	100	–	Ambiguous. The only way to properly express the correct number of significant figures in this situation is to express the number in scientific notation
	$1 \cdot 10^2$	–	1 significant figure
	$1.0 \cdot 10^2$	–	2 significant figures
	$1.00 \cdot 10^2$	–	3 significant figures
	100.	–	3 significant figures

2.7 Significant Figures in Calculations

When numbers are multiplied (or divided), the error in the accuracy of these numbers is also multiplied. The product of a multiplication typically has more numbers in it than are really significant. The following rules are used to determine the number of significant digits following various mathematical operations. When "truncating" large numbers to their proper number of significant figures, the rules for rounding off (next section) should be observed.

Multiplication and Division

Under multiplication and division, the final answer must have the <u>fewest number of significant figures</u> among the numbers involved in the calculation.

e.g.	24.015	×	6.50	=	156
	5 sig. figs.		3 sig. figs.		3 sig. figs.

Addition and Subtraction

Under addition and subtraction, the final answer must have <u>the fewest number of decimal places</u> among the numbers involved in the calculation.

e.g. $2.4015 \cdot 10^{-6} + 6.50 \cdot 10^{-4} = (0.024015 + 6.50) \cdot 10^{-4}$

6 decimal places   2 decimal places

$= 6.524015 \cdot 10^{-4}$ (answer should have has only 2 decimal places)

$= 6.52 \cdot 10^{-4}$

Final answer has only 2 decimal places

Note: Conversion factors are regarded as infinitely precise therefore should never be considered when the number of significant figures in the final answer is determined. For example, 2 moles of NaCl means "2." followed by an infinite amount of "0".

2.8 Rounding Off

In calculations, the number obtained usually has many more digits than would correctly express the precision to which the original measurements were made. Thus a calculated number must be reduced to the proper number of significant figures; this process is known as rounding off. The following set of rules represents one convention for rounding off numbers to the proper number of significant figures. In the examples that follow, the last appropriate significant digit is underlined.

First determine the correct number of significant figures to which the calculated number is to be rounded off. Inspect the digit immediately to the right of the last significant digit (N.B., in the examples that follow, the last valid significant digit is underlined).

If this digit is less than 5

Ignore the digit.

e.g. $0.0257\underline{5}39 = 0.0257\underline{5}$

If this digit is greater than or equal to 5

Increase the last significant digit by one.

e.g. $0.0299\underline{9}793 \rightarrow 0.0300\underline{0}$

Exercices:

1. How many significant figures are present in the following numbers?

   i. 1422
   ii. 65321
   iii. $1.004 \cdot 10^5$
   iv. 200
   v. 0.000100200

2. Round off numbers to 3 significant figures.

   i. 1566311
   ii. $2.7651 \cdot 10^{-3}$
   iii. $422.65 \cdot 10^{-5}$
   iv. 0.0011672
   v. 0.0101200

3. Perform the following calculations by hand and write the results with the appropriate number of significant figures.

   i. $234.56 - 1.002$
   ii. $3.45 \cdot 10^3 + 0.234 \cdot 10^3$
   iii. $3.45 \cdot 10^3 - 0.234 \cdot 10^3$
   iv. $6.27 \cdot 10^2 + 345.9$
   v. $1.2 \times 3.4589 \times 5.16$
   vi. $(2.34)^3$
   vii. 0234 / 0.0014
   viii. $56.78 + 26.51$
   ix. 83.29×4.56
   x. $47.8573 \times 7.53 + 87.5462 \times 2.5$

Answers

1. i. 4
   ii. 5
   iii. 4
   iv. ambiguous
   v. 6
2. i. 1.57×10^6
   ii. 2.77×10^{-3}
   iii. 4.23×10^{-3}
   iv. 1.17×10^{-3}
   v. 1.01×10^{-2}
3. i. 233.56
   ii. 3.68×10^3
   iii. 3.22×10^3
   iv. 9.73×10^2
   v. 21
   vi. 12.8
   vii. 1.7×10^5
   viii. 83.29
   ix. 3.80×10^2
   x. 5.82×10^2